A WAY TO IMPROVE THE USE OF CAS FOR INTEGRATION

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Background. Result of CAS integration or sometimes is not satisfactory [1], or sometimes even does not exist (despite the existence of the solution in the integral tables). For example in Mupad and Maple certain definite integral $I := \mathrm{int}(1/((1+x^2)(1+e^{2x})), x=-1..1);$ cannot solve. Until recently CAS only gave final result of integration. The new approach for CAS-supported integration includes interactive single-step computations (e.g. package "Analysis" in MuPad [2]; package "Student" in Mapel or Java user graphics interfaces called Maplets in Maple [3]).

Aim. By focusing on the issues of transparency of CAS techniques and their congruence with pencil versions [4], this study examines limitations of a step-by-step approach to CAS-supported integration and a way to reduce them.

Sources of Evidence. Interactive CAS integration enables shorter solutions. Consider the use of package "Student" in Maple and we give one example with different solutions of the same integration:

$$
\begin{array}{ll}
\int (x-x^2)^{-1/2}\,dx & \\
= \int \frac{2}{\sqrt{-4u^2+1}}\,du & [change, u = x - \frac{1}{2}, u] \\
= 2\int (-4u^2+1)^{-1/2}\,du & [const.multiple] \\
= 2\int \frac{1}{2}\,du_1 & [change, u = \frac{1}{2}\sin(u_1), u_1] \\
= u_1 & [const.] \\
= \arcsin(2u) & [revert] \\
= \arcsin(2x-1) & [revert]
\end{array}
$$

$$
\begin{array}{ll}
\int (x-x^2)^{-1/2}\,dx & \\
= \int 2\,dt & [change, x = \sin^2(t)] \\
= 2t & [const.] \\
= 2\arcsin(\sqrt{x}) & [revert]
\end{array}
$$

The first solution is given directly by Maple integration, and the second one is obtained manually by the input of substitution in the first step of the integration. Each such substitution which gives a different result or a result with fewer steps is an improvement of tools for integration. Introducing such substitutions, there is a possibility that CAS supported integration gives more solutions. The second example is previous certain definite integral $I := \mathrm{int}(1/((1+x^2)(1+e^{2x})), x=-1..1);$ which mentioned CAS tools cannot solve. The result is term $I = \pi/4$ and it is obtained by usaing transformation $1/((1+x^2)(1+e^{2x})) = 1/(1+x^2) - 1/((1+x^2)(1+e^{-2x}))$.

Main Argument. The previously mentioned transformation is an instance of general transformation of the form $\dfrac{1}{f(x)(1+e^{ax})} = \dfrac{1}{f(x)} - \dfrac{1}{f(x)(1+e^{-ax})}$, which enables solving implication:

$$I = \int_{-1}^{1} \frac{1}{f(x)(1+e^{ax})} dx \Rightarrow I = \frac{1}{2} \int_{-1}^{1} \frac{1}{f(x)} dx;$$

where $1/f(x)$ is an even integrable function over [-1,1] and $a \in R$. Using suitable transformations is vital to improving CAS integration.

Conclusion. In order to make CAS techniques transparent and congruent with their paper-and-pencil versions, an interactive step-by-step integration should be used. To obtain (more adequate) solutions by using this approach, new transformations at specific steps should be applied. While research should focus on uncovering these transformations, future CAS implementations should include them in their built-in features.

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